MA 125 6D, CALCULUS I

September 21, 2015

Name (Print last name first): ....

TEST I

Show all your work! No partial credit will be given for the answer only!

PART I

Part I consists of questions. Clearly write your answer in the space provided after each question. Show all of your your work!

All problems in Part I are 7 points each

Evaluate the following limits.

Question 1

Use the **definition** of the derivative to show that the derivative of $y = f(x) = x^2$ is $f'(x) = 2x$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} 2x+h$$

$$= \boxed{2x}$$

Question 2

Find the derivative of $f(x) = x\sin(x)$

Answer: ....

$$f'(x) = (x\sin x)'$$

$$= (x)'\sin x + x(\sin x)'$$

$$= \boxed{\sin x + x\cos x}$$

Question 7

Evaluate the limit $\lim_{x\to 3} \dfrac{x^2+2x-15}{x^2-9}$

$$= \lim_{x\to 3} \frac{(x-3)(x+5)}{(x-3)(x+3)} = \lim_{x\to 3} \frac{x+5}{x+3} = \frac{8}{6} = \boxed{\frac{4}{3}}$$

Answer: ....

PART II

Part II consists of 4 problems. You must show correct reasons to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 (10 points)

Suppose that $S(t) = t^3 - 3t$ ~~m~~ is the position of a particle at time t (in seconds) on a line. Find:

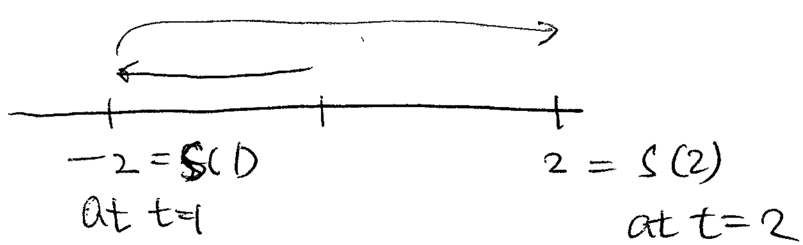
(a) the velocity at time t

(b) Draw a diagram up to $t = 2$

(c) the total distance traveled from $t = 0$ to $t = 2$.

(a) ~~at~~ $V(t) = S'(t) = 3t-3.$

(b)



(c) $2 + 4 = \underline{\underline{6}}.$

Question 3

Find the derivative of $y = f(x) = \frac{x^2}{1+x^2}$.

$$f'(x) = \left(\frac{x^2}{1+x^2}\right)' = \frac{2x(1+x^2) - x^2(2x)}{(1+x^2)^2}$$

$$= \frac{2x + 2x^3 - 2x^3}{(1+x^2)^2} = \boxed{\frac{2x}{(1+x^2)^2}}$$

Answer: ....

Question 4

Find the derivative of $y = f(x) = \sqrt[3]{x}(x + x^2 + x^3)$.

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}(x + x^2 + x^3) + x^{\frac{1}{3}}(1 + 2x + 3x^2)$$

$$= \frac{1}{3}x^{\frac{1}{3}} + \frac{1}{3}x^{\frac{4}{3}} + \frac{1}{3}x^{\frac{7}{3}} + x^{\frac{1}{3}} + 2x^{\frac{4}{3}} + 3x^{\frac{7}{3}}$$

4 Answer: ..7..............

Question 5

$$= \boxed{\frac{4}{3}x^{\frac{1}{3}} + \frac{7}{3}x^{\frac{4}{3}} + \frac{10}{3}x^{\frac{7}{3}}}$$

Find the equation of the tangent line to the graph of $y = f(x) = \tan(x)$ at the point $a = \pi/4$.

$$f'(x) = \sec^2 x \Rightarrow f(\tfrac{\pi}{4}) = \sec^2\tfrac{\pi}{4} = 2$$

$$f(\tfrac{\pi}{4}) = \tan(\tfrac{\pi}{4}) = 1$$

$$\boxed{\Rightarrow y = 2(x - \tfrac{\pi}{4}) + 1}$$

Answer: ....

Question 6

Use Squeeze Theorem to evaluate the limit $\lim_{x\to\infty} \frac{\sin(x)}{x}$

1. $-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$

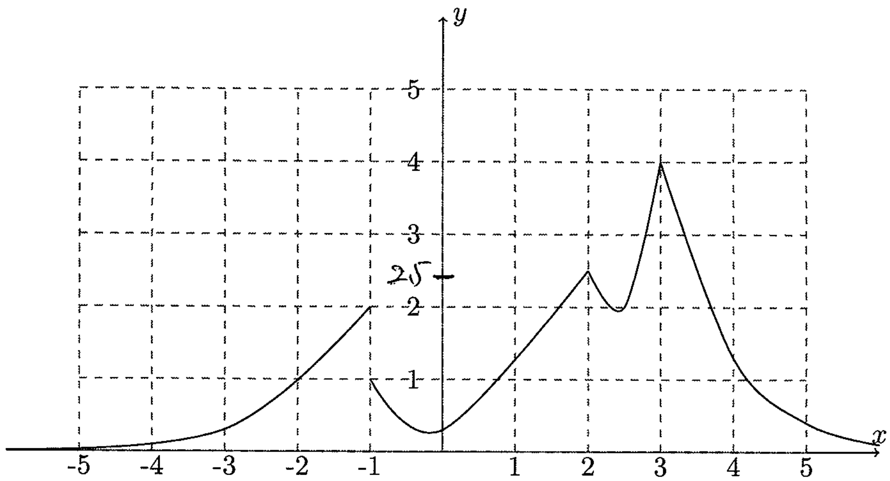
Answer: ....

2. $\lim_{x\to\infty}\left(-\frac{1}{x}\right) = \lim_{x\to\infty}\frac{1}{x} = 0$

$\Rightarrow$ By Squeeze Thm, $\lim_{x\to\infty}\frac{\sin x}{x} = 0$

Problem 2 (10 points)

Given the graph of the function $y = f(x)$ below find:



1. $\lim_{x\to -1^-} f(x) =$ 2
2. $\lim_{x\to -1^+} f(x) =$ 1
3. $\lim_{x\to -1} f(x) =$ does not exist
4. $\lim_{x\to 2^-} f(x) =$ 2.5
5. $\lim_{x\to 2^+} f(x) =$ 2.5
6. $\lim_{x\to 2} f(x) =$ 2.5
7. $\lim_{x\to \infty} f(x) =$ 0
8. $\lim_{x\to -\infty} f(x) =$ 0
9. State all intervals on which $f(x)$ is continuous. $(-\infty, -1), (-1, \infty)$
10. State all intervals where $f(x)$ is differentiable. $(-\infty, -1)\ (-1, 2)\ (2, 3)\ (3, \infty)$

Problem 3 (10 points)

Find all points on the graph of $f(x) = x^3 + 3x^2 + x$ where the tangent line is parallel to the line $y = 46x$, (Please denote a pair (a,f(a))).

1) $46 = \text{Slop} = f'(a)$

2) $f'(a) = 3a^2 + 6a + 1$

3). $3a^2 + 6a + 1 = 46 \Rightarrow 3a^2 + 6a - 45 = 0$

$\Rightarrow a^2 + 2a - 15 = 0 \Rightarrow (a-3)(a+5) = 0$

$\Rightarrow a = 3, \ a = -5.$

4) $f(3) = 27 + 27 + 3 = 57$

$f(-5) = (-5)^3 + 3(-5)^2 + (-5)$

$= -125 + 75 - 5 = -55$

$\therefore \boxed{(3, 57), \ (-5, -55)}$

Problem 4 (11 points)

Suppose $C(x) = 500 - 5x + 0.05x^2$ describes the cost (in 100's of dollars) from producing x (in thousands) items.

1. Given the definition of $C(x)$, what is the meaning of $C(200)$?

Total cost when producing 200 items

2. What is the meaning of $C'(200)$.

Marginal cost : ~~Average~~ rate of change of the cost when producing 200 items.

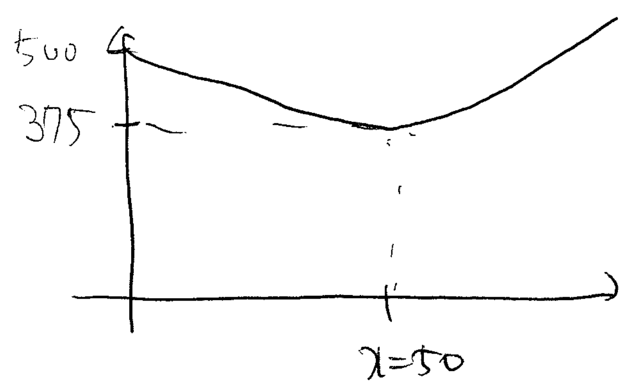
3. Compute $C'(x)$ for all $x \geq 0$. Determine where $C'(x) > 0$ and where $C'(x) < 0$.

$C'(x) = -5 + 0.1x = 0$

$\Rightarrow \underline{x = 50}. \Rightarrow$

$C'(x) < 0$	$0 < x < 50$
$C'(x) > 0$	$x > 50$

4. Use the above information to make a rough graph of $C(x)$ and decide how many items you should produce to minimize the cost.



$x = 50$

minimum cost

$\underline{375} = C(50)$.

Problem 5 (10 points)

Evaluate the following limits. Like always, justify your answers.

1. $\lim_{x\to\infty} \sqrt{x}+\sqrt{x+1}$ $= \infty + \infty = \infty$

$\boxed{= \infty}$

2. $\lim_{x\to\infty} \sqrt{x}-\sqrt{x+1}$ $= \infty - \infty$ not defined

we do not know now.

But $\lim_{x\to\infty} \sqrt{x}-\sqrt{x+1} = \lim_{x\to\infty} \frac{(\sqrt{x}-\sqrt{x+1})(\sqrt{x}+\sqrt{x+1})}{\sqrt{x}+\sqrt{x+1}}$

$= \lim_{x\to\infty} \frac{x-(x+1)}{\sqrt{x}+\sqrt{x+1}} = \lim_{x\to\infty} \frac{-1}{\sqrt{x}+\sqrt{x+1}} = \boxed{0.}$

Scratch paper